MATHEMATICS (860)

CLASS XII

*There will be **two** papers in the subject:*

***Paper I :** Theory (3 hours) ……80 marks*

***Paper II:** Project Work ……20 marks*

PAPER I (THEORY) – 80 Marks

*The syllabus is divided into **three** sections A, B and C.*

*Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **EITHER** Section B **OR** Section C.*

DISTRIBUTION OF MARKS FOR THE THEORY PAPER

S.No.	UNIT	TOTAL WEIGHTAGE
SECTION A: 65 MARKS		
1.	Relations and Functions	10 Marks
2.	Algebra	10 Marks
3.	Calculus	32 Marks
4.	Probability	13 Marks
SECTION B: 15 MARKS		
5.	Vectors	5 Marks
6.	Three - Dimensional Geometry	6 Marks
7.	Applications of Integrals	4 Marks
OR **SECTION C: 15 MARKS**		
8.	Application of Calculus	5 Marks
9.	Linear Regression	6 Marks
10.	Linear Programming	4 Marks
TOTAL		**80 Marks**

SECTION A

1. Relations and Functions

(i) Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, inverse of a function.

- *Relations as:*
  - *Relation on a set A*
  - *Identity relation, empty relation, universal relation.*
  - *Types of Relations: reflexive, symmetric, transitive and equivalence relation.*
- *Functions:*
  - *As special relations, concept of writing "y is a function of x" as y = f(x).*
  - *Types: one to one, many to one, into, onto.*
  - *Real Valued function.*
  - *Domain and range of a function.*
  - *Conditions of invertibility.*
  - *Invertible functions (algebraic functions only).*

(ii) Inverse Trigonometric Functions

Definition, domain, range, principal value branch. Elementary properties of inverse trigonometric functions.

- *Principal values.*
- *$sin^{-1}x$, $cos^{-1}x$, $tan^{-1}x$ etc*
- $sin^{-1}x = cos^{-1}\sqrt{1-x^2} = tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$.
- $sin^{-1}x = \text{cosec}^{-1}\dfrac{1}{\text{x}}$; $sin^{-1}x + cos^{-1}x = \dfrac{\pi}{2}$ *and similar relations for $cot^{-1}x$, $tan^{-1}x$, etc.*

$$sin^{-1}x \pm sin^{-1}y = sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$cos^{-1}x \pm cos^{-1}y = cos^{-1}\left(xy \mp \sqrt{1-y^2}\sqrt{1-x^2}\right)$$

similarly $tan^{-1}x + tan^{-1}y = tan^{-1}\dfrac{x+y}{1-xy}, xy < 1$

$$tan^{-1}x - tan^{-1}y = tan^{-1}\dfrac{x-y}{1+xy}, xy > -1$$

- *Formulae for $2sin^{-1}x$, $2cos^{-1}x$, $2tan^{-1}x$, $3tan^{-1}x$ etc. and application of these formulae.*

2. Algebra

Matrices and Determinants

(i) Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order upto 3). Invertible matrices and proof of the uniqueness of inverse, if it exists (here all matrices will have real entries).

(ii) Determinants

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

- *Types of matrices ($m \times n$; $m, n \leq 3$), order; Identity matrix, Diagonal matrix.*
- *Symmetric, Skew symmetric.*
- *Operation – addition, subtraction, multiplication of a matrix with scalar,*

multiplication of two matrices (the compatibility).

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = AB(say)$ *but* BA *is not possible.*

- *Singular and non-singular matrices.*
- *Existence of two non-zero matrices whose product is a zero matrix.*
- *Inverse* $(2\times2,\ 3\times3)$ $A^{-1} = \dfrac{AdjA}{|A|}$

- *Martin's Rule (i.e. using matrices)*

  $a_1x + b_1y + c_1z = d_1$

  $a_2x + b_2y + c_2z = d_2$

  $a_3x + b_3y + c_3z = d_3$

  $$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

  $AX = B \Rightarrow X = A^{-1}B$

  Problems based on above.

  NOTE: *The conditions for consistency of equations in two and three variables, using matrices, are to be covered.*

- *Determinants*
  - *Order.*
  - *Minors.*
  - *Cofactors.*
  - *Expansion.*
  - *Applications of determinants in finding the area of triangle and collinearity.*
  - *Properties of determinants. Problems based on properties of determinants.*

3. Calculus

(i) Continuity, Differentiability and Differentiation. Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

- *Continuity*
  - *Continuity of a function at a point* $x = a$.
  - *Continuity of a function in an interval.*
  - *Algebra of continues function.*
  - *Removable discontinuity.*
- *Differentiation*
  - *Concept of continuity and differentiability of* $|x|$, $[x]$, *etc.*
  - *Derivatives of trigonometric functions.*
  - *Derivatives of exponential functions.*
  - *Derivatives of logarithmic functions.*
  - *Derivatives of inverse trigonometric functions - differentiation by means of substitution.*
  - *Derivatives of implicit functions and chain rule.*
  - *Derivatives of Parametric functions.*
  - *Differentiation of a function with respect to another function e.g. differentiation of* $\sin x^3$ *with respect to* x^3.
  - *Logarithmic Differentiation - Finding dy/dx when* $y = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$.
  - *Successive differentiation up to 2nd order.*

NOTE: *Derivatives of composite functions using chain rule.*

(ii) Applications of Derivatives

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

- *Equation of Tangent and Normal*
- *Rate measure.*
- *Increasing and decreasing functions.*
- *Maxima and minima.*
  - *Stationary/turning points.*
  - *Absolute maxima/minima*
  - *local maxima/minima*
  - *First derivatives test and second derivatives test*
  - *Application problems based on maxima and minima.*

(iii) Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

- *Indefinite integral*
  - *Integration as the inverse of differentiation.*
  - *Anti-derivatives of polynomials and functions* $(ax+b)^n$ *, sinx, cosx,* sec^2x*,* $cosec^2x$ *etc .*
  - *Integrals of the type* sin^2x*,* sin^3x*,* sin^4x*,* cos^2x*,* cos^3x*,* cos^4x*.*
  - *Integration of* $1/x$*,* e^x*.*
  - *Integration by substitution.*
  - *Integrals of the type* $f'(x)[f(x)]^n$*,* $\frac{f'(x)}{f(x)}$.
  - *Integration of tanx, cotx, secx, cosecx.*
  - *Integration by parts.*
  - *Integration using partial fractions.*

    Expressions of the form $\frac{f(x)}{g(x)}$ *when degree of f(x) < degree of g(x)*

E.g. $$\frac{x+2}{(x-3)(x+1)}=\frac{A}{x-3}+\frac{B}{x+1}$$

$$\frac{x+2}{(x-2)(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

$$\frac{x+1}{(x^2+3)(x-1)}=\frac{Ax+B}{x^2+3}+\frac{C}{x-1}$$

When degree of f (x) ≥ degree of g(x),

e.g. $$\frac{x^2+1}{x^2+3x+2}=1-\left(\frac{3x+1}{x^2+3x+2}\right)$$

- *Integrals of the type:*

$$\int\frac{dx}{x^2\pm a^2},\int\frac{dx}{\sqrt{x^2\pm a^2}},\int\frac{px+q}{ax^2+bx+c}dx,\int\frac{px+q}{\sqrt{ax^2+bx+c}}dx$$

and $\int\sqrt{a^2\pm x^2}\ dx, \int\sqrt{x^2-a^2}\ dx$,

$$\int\sqrt{ax^2+bx+c}\ dx,\ \int(px+q)\sqrt{ax^2+bx+c}\ dx,$$

integrations reducible to the above forms.

$$\int\frac{dx}{a\cos x+b\sin x},$$

$$\int\frac{dx}{a+b\cos x},\int\frac{dx}{a+b\sin x}\int\frac{dx}{a\cos x+b\sin x+c},$$

$$\int\frac{(a\cos x+b\sin x)dx}{c\cos x+d\sin x},$$

$$\int\frac{dx}{a\cos^2x+b\sin^2x+c}$$

$$\int\frac{1\pm x^2}{1+x^4}dx,$$

$$\int\frac{dx}{1+x^4},\int\sqrt{\tan x}dx,\int\sqrt{\cot x}dx\ \ etc.$$

- *Definite Integral*
  - *Fundamental theorem of calculus (without proof)*
  - *Properties of definite integrals.*
  - *Problems based on the following properties of definite integrals are to be covered.*

$$\int_a^b f(x)dx = \int_a^b f(t)dt$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

where $a < c < b$

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if \quad f(2a-x) = f(x) \\ 0, \quad f(2a-x) = -f(x) \end{cases}$$

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, \text{if } f \text{ is an even function} \\ 0, \text{if } f \text{ is an odd function} \end{cases}$$

(iv) Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type: $\frac{dy}{dx}$+py= q, where p and q are functions of x or constants. $\frac{dx}{dy}$+ px = q, where p and q are functions of y or constants.

- *Differential equations, order and degree.*
- *Formation of differential equation by eliminating arbitrary constant(s).*
- *Solution of differential equations.*
- *Variable separable.*
- *Homogeneous equations.*
- *Linear form* $\frac{dy}{dx} + Py = Q$ *where P and Q are functions of x only. Similarly, for dx/dy.*

NOTE 1: *Equations reducible to variable separable type are included.*

NOTE 2: *The second order differential equations are excluded.*

4. **Probability**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

- *Independent and dependent events conditional events.*
- *Laws of Probability, addition theorem, multiplication theorem, conditional probability.*
- *Theorem of Total Probability.*
- *Baye's theorem.*
- *Theoretical probability distribution, probability distribution function; mean of random variable.*

SECTION B

5. **Vectors**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

- *As directed line segments.*
- *Magnitude and direction of a vector.*
- *Types: equal vectors, unit vectors, zero vector.*
- *Position vector.*
- *Components of a vector.*
- *Vectors in two and three dimensions.*
- $\hat{i}, \hat{j}, \hat{k}$ *as unit vectors along the x, y and the z axes; expressing a vector in terms of the unit vectors.*
- *Operations: Sum and Difference of vectors; scalar multiplication of a vector.*
- *Section formula.*
- *Scalar (dot) product of vectors and its geometrical significance.*
- *Cross product - its properties - area of a triangle, area of parallelogram, collinear vectors.*

NOTE: Proofs of geometrical theorems by using Vector algebra are excluded.

6. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

- *Equation of x-axis, y-axis, z axis and lines parallel to them.*
- *Equation of xy - plane, yz – plane, zx – plane.*
- *Direction cosines, direction ratios.*
- *Angle between two lines in terms of direction cosines /direction ratios.*
- *Condition for lines to be perpendicular/ parallel.*

- *Lines*
  - *Cartesian and vector equations of a line through one and two points.*
  - *Coplanar and skew lines.*
  - *Conditions for intersection of two lines.*
  - *Distance of a point from a line.*
  - *Shortest distance between two lines.*
- *Planes*
  - *Cartesian and vector equation of a plane.*
  - *Direction ratios of the normal to the plane.*
  - *One point form.*
  - *Normal form.*
  - *Intercept form.*
  - *Distance of a point from a plane.*
  - *Intersection of the line and plane.*
  - *Angle between two planes, a line and a plane.*

7. Application of Integrals

Application in finding the area bounded by simple curves and coordinate axes. Area enclosed between two curves.

- *Application of definite integrals - area bounded by curves, lines and coordinate axes is required to be covered.*
- *Simple curves: lines, circles/ parabolas/ ellipses, polynomial functions, modulus function.*

SECTION C

8. Application of Calculus

Application of Calculus in Commerce and Economics in the following:

- Cost function,
- average cost,
- marginal cost and its interpretation
- demand function,
- revenue function,
- marginal revenue function and its interpretation,
- Profit function and breakeven point.
- Rough sketching of the following curves: AR, MR, R, C, AC, MC and their mathematical interpretation using the concept of maxima & minima and increasing-decreasing functions.

Self-explanatory

NOTE: Application involving differentiation, increasing and decreasing function and maxima and minima to be covered.

9. Linear Regression

- Lines of regression of x on y and y on x.
- Scatter diagrams
- The method of least squares.
- Lines of best fit.
- Regression coefficient of x on y and y on x.
- $b_{xy} \times b_{yx} = r^2$, $0 \le b_{xy} \times b_{yx} \le 1$
- Identification of regression equations
- Properties of regression lines.
- Estimation of the value of one variable using the value of other variable from appropriate line of regression.

Self-explanatory

10. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Introduction, definition of related terminology such as constraints, objective function, optimization, advantages of linear programming; limitations of linear programming; application areas of linear programming; different types of linear programming (L.P.) problems, mathematical formulation of L.P problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimum feasible solution.

PAPER II – PROJECT WORK – 20 Marks

Candidates will be expected to have completed **two** projects, one from Section A and one from ***either*** Section B **or** Section C.

The project work will be assessed by the subject teacher and a Visiting Examiner appointed locally and approved by the Council.

Mark allocation for **each** Project [10 marks]:

Overall format	1 mark
Content	4 marks
Findings	2 marks
Viva-voce based on the Project	3 marks
Total	**10 marks**

List of suggested assignments for Project Work:

Section A

1. Using a graph, demonstrate a function which is one-one but not onto.
2. Using a graph demonstrate a function which is invertible.
3. Construct a composition table using a binary function addition/multiplication modulo upto 5 and verify the existence of the properties of binary operation.
4. Draw the graph of $y = \sin^{-1} x$ (or any other inverse trigonometric function), using the graph of $y = \sin x$ (or any other relevant trigonometric function). Demonstrate the concept of mirror line (about $y = x$) and find its domain and range.
5. Explore the principal value of the function $\sin^{-1} x$ (or any other inverse trigonometric function) using a unit circle.
6. Find the derivatives of a determinant of the order of 3 x 3 and verify the same by other methods.
7. Verify the consistency of the system of three linear equations of two variables and verify the same graphically. Give its geometrical interpretation.
8. For a dependent system (non-homogeneous) of three linear equations of three variables, identify infinite number of solutions.
9. For a given function, give the geometrical interpretation of Mean Value theorems. Explain the significance of closed and open intervals for continuity and differentiability properties of the theorems.
10. Explain the concepts of increasing and decreasing functions, using geometrical

significance of *dy/dx*. Illustrate with proper examples.

11. Explain the geometrical significance of point of inflexion with examples and illustrate it using graphs.
12. Explain and illustrate (with suitable examples) the concept of local maxima and local minima using graph.
13. Explain and illustrate (with suitable examples) the concept of absolute maxima and absolute minima using graph.
14. Illustrate the concept of definite integral $\int_a^b f(x)\,dx$, expressing as the limit of a sum and verify it by actual integration.
15. Demonstrate application of differential equations to solve a given problem (example, population increase or decrease, bacteria count in a culture, etc.).
16. Explain the conditional probability, the theorem of total probability and the concept of Bayes' theorem with suitable examples.
17. Explain the types of probability distributions and derive mean and variance of binomial probability distribution for a given function.

Section B

18. Using Vector algebra, find the area of a parallelogram/triangle. Also, derive the area analytically and verify the same.
19. Using Vector algebra, prove the formulae of properties of triangles (sine/cosine rule, etc.)
20. Using Vector algebra, prove the formulae of compound angles, e.g. sin (A + B) = Sin A Cos B + Sin B Cos A, etc.
21. Describe the geometrical interpretation of scalar triple product and for a given data, find the scalar triple product.
22. Find the image of a line with respect to a given plane.
23. Find the distance of a point from a given plane measured parallel to a given line.
24. Find the distance of a point from a line measured parallel to a given plane.
25. Find the area bounded by a parabola and an oblique line.
26. Find the area bounded by a circle and an oblique line.
27. Find the area bounded by an ellipse and an oblique line.
28. Find the area bounded by a circle and a circle.
29. Find the area bounded by a parabola and a parabola.
30. Find the area bounded by a circle and a parabola.

(Any other pair of curves which are specified in the syllabus may also be taken.)

Section C

31. Draw a rough sketch of Cost (C), Average Cost (AC) and Marginal Cost (MC)

    Or

    Revenue (R), Average Revenue (AR) and Marginal Revenue (MR).

    Give their mathematical interpretation using the concept of increasing - decreasing functions and maxima-minima.
32. For a given data, find regression equations by the method of least squares. Also find angles between regression lines.
33. Draw the scatter diagram for a given data. Use it to draw the lines of best fit and estimate the value of Y when X is given and vice-versa.
34. Using any suitable data, find the minimum cost by applying the concept of Transportation problem.
35. Using any suitable data, find the minimum cost and maximum nutritional value by applying the concept of Diet problem.
36. Using any suitable data, find the Optimum cost in the manufacturing problem by formulating a linear programming problem (LPP).

NOTE: No question paper for Project Work will be set by CISCE.

SAMPLE TABLE FOR PROJECT WORK

S. No.	Unique Identification Number (Unique ID) of the candidate	**PROJECT 1**					**PROJECT 2**					TOTAL MARKS
		A	B	C	D	E	F	G	H	I	J	
		Teacher	Visiting Examiner	Average Marks (A + B ÷ 2)	Viva-Voce by Visiting Examiner	Total Marks (C + D)	Teacher	Visiting Examiner	Average Marks (F + G ÷ 2)	Viva-Voce by Visiting Examiner	Total Marks (H + I)	(E + J)
		7 Marks*	7 Marks*	7 Marks	3 Marks	10 Marks	7 Marks*	7 Marks*	7 Marks	3 Marks	10 Marks	20 Marks
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												

*Breakup of 7 Marks to be awarded separately by the Teacher and the Visiting Examiner is as follows:		Name of Teacher: Signature: Date
Overall Format	1 Mark	
Content	4 Marks	Name of Visiting Examiner Signature: Date
Findings	2 Marks	

NOTE: VIVA-VOCE (3 Marks) for each Project is to be conducted only by the Visiting Examiner, and should be based on the Project only